

Bouncing Anisotropic Universes with Varying Constants

John D. Barrow^{1*} and David Sloan^{1†}

¹ DAMTP, Centre for Mathematical Sciences, Wilberforce Rd.,
Cambridge University, Cambridge CB3 0WA, UK

We examine the evolution of a closed, homogeneous and anisotropic cosmology subject to a variation of the fine structure 'constant', α within the context of the theory introduced by Bekenstein, Sandvik, Barrow and Magueijo, which generalises Maxwell's equations and general relativity. The variation of α permits an effective ghost scalar field, whose negative energy density becomes dominant at small length scales, leading to a bouncing cosmology. A thermodynamically motivated coupling which describes energy exchange between the effective ghost field and the radiation field leads to an expanding, isotropizing sequence of bounces. In the absence of entropy production we also find solutions with stable anisotropic oscillations around a static universe.

PACS numbers: 98.80.-k, 06.20.Jr, 04.20.-q

I. INTRODUCTION

Spatially homogeneous cosmological models are a key area of study within relativity. The introduction of anisotropies gives rise to models in which a richer dynamical structure emerges, yet the cosmology remains simple enough to provide analytic and simple numerical results. These models serve as a test-bed for physical theories, and allow us analyse questions about why the universe appears to be highly isotropic, whether inflation occurs for generic or stable sets of initial data, the effects of anisotropy on astronomical observables, and the behaviour of cosmological models on approach to spacetime singularities [1], [2], [3].

The idea that the fine structure constant, α , is a spacetime varying scalar field was first investigated by Bekenstein [4], who created a natural generalisation of Maxwell's equations to accommodate a varying electron charge. This idea was extended to include gravity and provide a theory to explore cosmological consequences of varying α by Sandvik et al [5]. The resulting Bekenstein-Sandvik-Barrow-Magueijo (BSBM) isotropic cosmological models were found and used in conjunction with the astronomical data on varying α obtained from observations of high redshift quasar spectra [6].

More recently, the BSBM theory has been extended to included the case where there is a coupling function (rather than simply a coupling constant) between the charged matter fields and the scalar field driving changes in α [7] and where that scalar field possesses a self-interaction potential [8]. These theories are the analogues of the Jordan-Brans-Dicke theories for varying G [9].

In [10] it was shown how theories of this type could produce singularity-free homogeneous and isotropic cosmologies which displayed stable oscillations around an Einstein static universe because the effect of variations in the scalar field driving variations in α is to introduce a negative 'ghost' density. Barrow and Tsagas [11] considered a broader context for these solutions and showed how the inclusion of simple anisotropic expansion can modify the results because the anisotropy can diverge just as quickly as a bounce-producing ghost scalar field on a approach to the singularity. In this paper we will consider more general closed anisotropic cosmologies with anisotropic 3-curvature in this same context.

Matter bounces introduced by the presence of ghost fields are not a new discovery (for a detailed examination see [12]). However, in BSBM models the ghost field is an effective manifestation of underlying physics, not a new matter source introduced by hand. In such models quantum effects are ignored because of the

*Electronic address: J.D.Barrow@damtp.cam.ac.uk

†Electronic address: djs228@hermes.cam.ac.uk

prevailing attitude that ghost fields should not be quantized (and are in fact ill-behaved when quantized, with negative probability states). Furthermore, any coupling between a ghost field and a non-ghost field would allow an infinite amount of energy to be transferred from the ghost field.

We simply take the view that BSBM models can serve as test models for bouncing cosmologies. The idea of a “phoenix” universe within relativity is almost as old as big-bang models themselves, and goes back to Tolman [13] and Lemaître [14]. This classical picture of oscillating closed universes with zero value of the cosmological constant, Λ , painted by Tolman is well known. If there is no entropy production then cycles for the time-evolution of the scale factor are periodic with the same amplitude and total lifetime. If entropy increase is introduced in accord with the second law of thermodynamics then the oscillating cycles become larger and longer to the infinite future. The classical picture for isotropic universes was competed by Barrow and Dąbrowski [15], who showed that if a positive cosmological constant is included then the sequence of growing cycles will always come to an end, no matter how small the value of $\Lambda > 0$. The ensuing behaviour will be to approach de Sitter expansion. If the entropy increase from cycle to cycle is small then the asymptotic state will be one in which the expansion is very close to a zero-curvature state with comparable energy densities associated with matter and dark energy (ie the cosmological constant). The dark energy will necessarily be slightly dominant and the curvature will be positive – not unlike the situation in our observed universe. Barrow and Dąbrowski [15] also considered the evolution of some simple bouncing anisotropic universes of Kantowski-Sachs type, but not in a context that included varying constants.

Many current quantum theories of gravity exhibit curvature singularity avoidance, often in the form of a bounce (although they do not necessarily avoid geodesic incompleteness). In Loop Quantum Cosmology, holonomy corrections to the Friedmann equation give rise to a bounce at Planck scales (see [16] for a review). Horava-Lifschitz gravity also introduces higher-

order curvature corrections to Einstein’s equations which can cause the universe to bounce [17] for some parameter choices. In the latter case the dynamics of an anisotropic solution have also been explored [18].

The aim of this paper is to extend [10] and [15] to spatially homogeneous models which exhibit local rotational symmetry (LRS) and so are effectively axisymmetric. LRS models exhibit some of the features of full anisotropic model [19], yet the differential equations governing their dynamics can be solved with relative ease using numerical solvers and exact methods. We will begin in section II by setting out the action principle underlying variation of the fine structure ‘constant’, then in section III we set out the equations of motion for our system and define quantities of physical interest, such as shear and Hubble expansion rates, in terms of metric variables. Section IV deals with two specific solutions to the equations of motion: a static solution and ghost-induced inflation. In particular, we will focus on the role played by anisotropies in both these cases and examine perturbations about isotropic cases.

II. THEORIES OF VARYING ALPHA

Varying “constants” can be described by extensions of the standard model of particle physics and/or general relativity (GR) by the promotion of constants to space and time dependent scalar fields. A well known and much studied example is that of Jordan-Brans-Dicke theory in which GR is extended by generalising Newton’s constant G to become a field variable [9]. These self-consistent models for the variation of constants necessarily contain conservation equations for the energy and momentum carried by the varying scalar field and the gravitational field equations account for the scalar field’s effect on the spacetime geometry. This is in contrast to much of the old literature on varying constants, other than G , which merely ‘write-in’ variations of constants into the equations that hold in the theory where the constant does not vary. The existence of a self-consistent theory for the variation of a constant also shows

that much discussion about the meaning of the variation of dimensional constants is not relevant because the solution of the second-order conservation equation for the scalar field describing the variation of a traditional constant always produces constants of integration with the same dimensions as the varying constant and a dimensionless combination is trivially available.

Physical models with extra dimensions often exhibit massless or light degrees of freedom which can lead to the variation of such constants [20], [21] and there is the possibility for observational bounds to be placed on any shift in the size of extra dimensions over the age of the universe [22],[23].

In this paper we shall follow the BSBM model in which the fine structure constant, α is taken to be dynamical. Evidence of a dipolar spatial variation has been recently claimed [6] and therefore it is natural to extend this scenario to consider space and time variations but in this paper we will only discuss time variations so that we can confine attention to ordinary differential equations. Such variations are bounded by terrestrial experiments to have small variation at present [24] [25]. However evidence that the variation is small currently does not rule out more significant changes in the past. In particular, in BSBM theories α is not expected to vary during the radiation era, to increase only logarithmically in time during the cold dark matter dominated era, and then to become constant after the universal expansion begins accelerating. Thus laboratory experiments today would not be expected to find evidence for the variation of α found in high-redshift quasar observations (that derive from epochs before the universe began accelerating) even though it has been proved that any cosmological variations in α will be seen in terrestrial experiments [26].

The BSBM model describes the effect of varying the fine structure constant by the introduction of a scalar dielectric field, ψ with evolution of the charge of an electron given by $e = e_0 e^\psi$, in which e_0 is the value of the electron charge at some fixed time, for example today. Notice that e_0 is a fundamental constant and e/e_0 is dimensionless. It has been shown [27] that in spite of modifying black hole solutions, the variability

of α respects the second law of thermodynamics. This will be important in section III, as we will assume that all couplings between our fields obey the second law.

The physical action is given by

$$S = \int \sqrt{-g} (\mathcal{L}_g + \mathcal{L}_m + L_\psi + e^{-2\psi} \mathcal{L}_{em}) \quad (2.1)$$

where $\mathcal{L}_g = R/16\pi G$ is the usual Einstein-Hilbert Lagrangian, $\mathcal{L}_\psi = -\frac{\omega}{2} \partial_u \psi \partial^u \psi$ governs the scalar dielectric field, ψ , $\mathcal{L}_{em} = -\frac{1}{4} f_{\mu\nu} f^{\mu\nu}$, and \mathcal{L}_m is a matter Lagrangian independent of ψ . Of particular importance to this paper is the constant coupling parameter ω which we shall take to be negative, so rendering ψ an effective ghost scalar field. We do not consider the generalised case where $\omega'(\psi) \neq 0$, see [7]. From now on it will be convenient to simply consider the effective field, rather than the underlying dielectric. This field is massless, and it is clear from the action that its motion will be monotonic, as will be that of the effective induced fine structure constant. In terms of fluids, this field will appear to be stiff, with equation of state $p_\psi = \rho_\psi < 0$. In a closed anisotropic cosmological model we will allow energy exchange to occur between the ψ field and an equilibrium radiation field with equation of state $3p_r = \rho_r$, to model an entropy increasing non-equilibrium process.

III. COSMOLOGICAL EXPANSION

The physical system under consideration will consist of a homogeneous anisotropic cosmology. For simplicity we will examine a system which is locally rotationally symmetric, and use this to gain insight into the more general case. For a concise review of these cosmological models, see [19]. This model is general enough to contain the purely general relativistic ingredient of anisotropic 3-curvature, which is missing from the simple anisotropic models of Bianchi types I and V. It includes the closed Bianchi type IX universe but only in the axisymmetric case where no chaotic behaviour occurs. The LRS type IX metric is

$$ds^2 = dt^2 - h_{ij} \sigma^i \sigma^j$$

where σ^i are the $SO(3)$ invariant 1-forms [28]

$$\begin{aligned}\sigma^1 &= \cos \psi d\theta + \sin \psi \sin \theta d\phi \\ \sigma^2 &= -\sin \psi d\theta + \cos \psi \sin \theta d\phi \\ \sigma^3 &= d\psi + \cos \theta d\phi\end{aligned}$$

and the LRS condition requires

$$h_{ij} = \text{diag}\{a(t), b(t), b(t)\}$$

The metric contains two time-dependent scale factors (due to the LRS condition), $a(t)$ and $b(t)$. The energy densities are denoted by ρ_r for radiation, ρ_ψ for the scalar field, and ρ_Λ for the cosmological constant, and the total density and pressure are ρ and p , where

$$\rho = \rho_r + \rho_\psi + \rho_\Lambda.$$

All matter matter sources have isotropic pressures. The independent variables are the principal 3-curvatures

$$\begin{aligned}R_1^1 &= \frac{a^2}{2b^4}, \\ R_2^2 &= \frac{1}{b^2} - \frac{a^2}{2b^4};\end{aligned}$$

the mean Hubble expansion rate is defined by

$$H = \frac{1}{3}(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b}),$$

and the expansion shear scalar by

$$\sigma = \frac{1}{3}(\frac{\dot{b}}{b} - \frac{\dot{a}}{a})$$

These variables are subject to a constraint equation (the generalized Friedmann equation with $8\pi G = c = 1$)

$$\rho = \frac{1}{b^2} - \frac{a^2}{4b^4} + 2\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2}. \quad (3.1)$$

The remaining field equations are:

$$\frac{\ddot{a}}{a} = -\frac{1}{2b^4} - 2\frac{\dot{a}\dot{b}}{ab} + \frac{\rho - p}{2}, \quad (3.2)$$

$$\frac{\ddot{b}}{b} = \frac{a^2}{2b^4} - \frac{1}{b^2} - \frac{\dot{a}\dot{b}}{ab} - \frac{\dot{b}^2}{b^2} + \frac{\rho - p}{2}, \quad (3.3)$$

$$\dot{\sigma} = -3H\sigma + \frac{1}{3}(R_1 - R_2), \quad (3.4)$$

$$\dot{H} = -H^2 - 2\sigma^2 - \frac{1}{6}(\rho + 3p). \quad (3.5)$$

These reduce to the special case of the closed Friedmann universes when $a = b$. The shear does not evolve with $\sigma \propto (ab^2)^{-1}$ as in Bianchi type I models because of the 3-curvature anisotropy on the right-hand side of eq. (3.4). For ease of exposition, let us define the variables $r = \frac{a}{b}$ and $H_b = \dot{b}/b$. The essential field equations then simplify to

$$\frac{\ddot{r}}{r} = \frac{1 - r^2}{b^2} - \frac{3\dot{r}\dot{b}}{rb}, \quad (3.6)$$

$$\sigma = -\frac{\dot{r}}{3r}, \quad (3.7)$$

$$H = \frac{\dot{b}}{b} - 3\sigma = H_b - 3\sigma. \quad (3.8)$$

The continuity equation, which implies constraint conservation, is

$$\dot{\rho} + 3H(\rho + p) = 0. \quad (3.9)$$

This governs the total energy density and pressure. Now, we introduce some energy exchange between the fluids so that they obey

$$\dot{\rho}_\psi + 6H\rho_\psi = s, \quad (3.10)$$

$$\dot{\rho}_r + 4H\rho_r = -s, \quad (3.11)$$

where, s parametrises the flow of energy between the scalar field and radiation. For our purposes, this will be taken to be of the form

$$s = -\rho_\psi \beta$$

where

$$\beta = \beta_0 + \beta_H H^2 + \beta_\sigma \sigma^2$$

can include a linear coupling β_0 , plus possible bulk, β_H , and shear, β_σ , viscous contributions. In general, the β 's need not be constants.

The scalar dielectric field evolves according to

$$\ddot{\psi} + (3H + \beta)\dot{\psi} = 0 \quad (3.12)$$

and so

$$\dot{\psi} \propto a^{-3} \exp[-\int \beta dt].$$

IV. SOLUTIONS

In what follows we consider the Bianchi IX case with no cosmological constant ($\rho_\Lambda = 0$). If the matter content is a perfect fluid and obeys $\rho + 3p > 0$, then these universes expand from an initial curvature singularity to a maximum size before collapsing back to a future curvature singularity; the spacetime is past and future geodesically incomplete. However, since the effect of varying the fine structure constant is to produce a ghost scalar field with $\rho_\psi < 0$ which will dominate dynamics at small length scales, the energy condition is violated and solutions to the BSBM model exist which have infinite past and future temporal range.

In this section we will examine two particular solutions to the equations of motion. The first is that of a static spacetime, and its behaviour under perturbations. The second is that of an spacetime in which the coupling between fields leads to inflationary behaviour.

A. The Static Solution

There exists a static solution of the form

$$\rho_\psi = -\frac{3}{4b^2} \quad \rho_r = \frac{3}{2b^2}, \quad (4.1)$$

for any given value of b . Note that in order to be static the solution must be isotropic ($\sigma = 0$), since from eq. (3.4) the 3-curvatures must match ($R_1^1 = R_2^2$) for the shear to remain constant and eq. (3.8) requires $\sigma = 0$.

Now consider making a small perturbation about the isotropic solution by introducing a small anisotropy: $r = 1 + \epsilon$ where $r = 1$ represents isotropy. From 3.6, we find that

$$\ddot{\epsilon} = -3\epsilon H_b + \frac{2\epsilon + 3\epsilon^2 + \epsilon^3}{b^2}. \quad (4.2)$$

Without loss of generality, we take the unperturbed static solution to have $b = 1$. If we introduce small parameters $\delta(t)$ and $\eta(t)$ such that $b = 1 + \eta$, and $\rho = 3/2 + \delta$, then to first order in our small parameters:

$$\epsilon(t) = \epsilon_0 \sin(\sqrt{2}t). \quad (4.3)$$

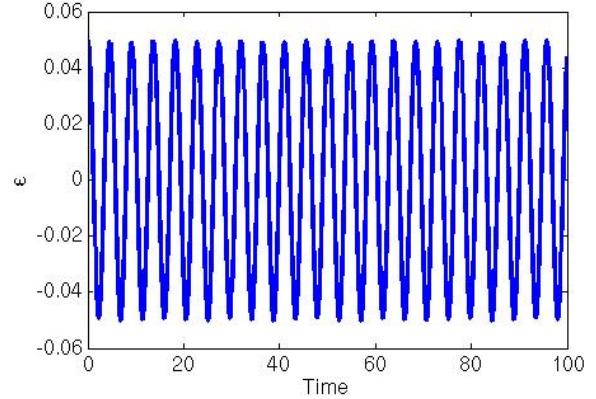


FIG. 1: ϵ versus time with no coupling ($s = 0$). Initial values: $r = 1.05, \sigma = 0, H = 0$.

In the case where there is no coupling between the fields ($s = 0$), these oscillations continue endlessly as shown in figure 1. However, once coupling is introduced ($s \neq 0$), the static case becomes unstable because the balance between ρ_ψ and ρ_r is broken, energy is transferred from the ghost field that supports stable oscillations and eventually, after several oscillations, it settles into radiation-dominated expansion, shown in figure 2:

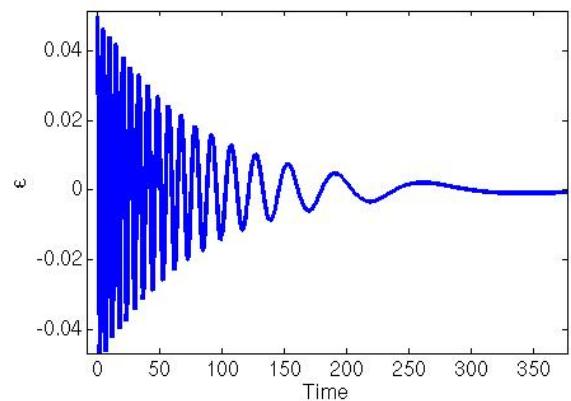


FIG. 2: ϵ versus time with coupling turned on, $s \neq 0$, showing the system isotropizing. Initial values: $r = 1.05, \sigma = 0, H = 0, \beta_0 = 0.05, \beta_H = \beta_\sigma = 0$.

From the constraint equation 3.1, we find that to first order, our small parameters are related by $2\delta = -\epsilon - 3\eta$. Furthermore, we can

decompose δ into the radiation and scalar field components, δ_r and δ_ψ . In the absence of field couplings, there is a relationship between these fields, due to their coupled equations of motion 3.12, 3.10:

$$\rho_r \propto \rho_\psi^{3/2}. \quad (4.4)$$

In the static case under consideration, the constant of proportionality is $-\sqrt{6}$. Note that this relationship is broken by introducing a coupling between the fields. For small δ we are therefore led to $\delta_r = 4\delta = -2\epsilon - 6\eta$.

From 3.3, the evolution of η is given by:

$$\ddot{\eta} = \epsilon - \eta + \frac{\delta_r}{3} = \frac{\epsilon}{3} - \eta. \quad (4.5)$$

Therefore there is a (more complicated) stable oscillatory behaviour for η about the static solution and the evolution of ϵ has already been determined by 4.3. Thus we have an unusual behaviour characterised by stable anisotropic oscillations around the isotropic Einstein static universe. This generalises the simple isotropic oscillations about the static universe that exist in Friedmann universes with a ghost field found in [10] and [11].

We can now determine further effects of allowing a coupling between the fields. The second law of thermodynamics requires $s \geq 0$. The exact form of s - taking into account terms representing constant coupling, bulk and shear viscosities, will of course affect the exact dynamics. However, it is possible to make progress by assuming only that s is non-negative.

First consider the case of no coupling ($s = 0$). The evolution of the radiation field is determined by

$$\frac{\dot{\rho}_r}{\rho_r} = -\frac{4}{3}H = -\frac{4}{3}\left(\frac{\dot{\epsilon}}{3} + \dot{\eta}\right). \quad (4.6)$$

Due to the cyclic behaviour of ϵ and η (and hence of their derivatives), δ_r will also cycle, returning to its initial value, as any integral of the right-hand side of 4.6 across a complete cycle will be zero. However, with a positive coupling between the fields, this relationship is broken, and

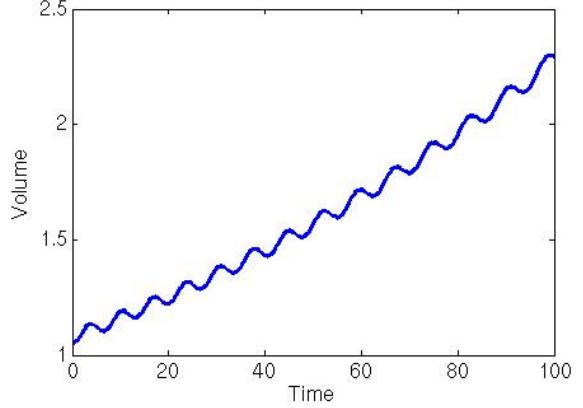


FIG. 3: Spatial volume versus time with coupling turned on ($s \neq 0$). The system oscillates, but expands from one cycle to the next. Initial values: $r = 1.05, \sigma = 0, H = 0, \beta_0 = 0.01, \beta_H = \beta_\sigma = 0$.

a term which is always non-negative (and so has a positive integral across cycles) must be added. Hence, across a cycle in η and ϵ , the value of δ_r now increases and we must adjust our equation of motion 4.5 to include this term. We write

$$\ddot{\eta} = \epsilon + \eta + \frac{\delta_r}{3} = \frac{\epsilon}{3} - \eta + \Delta, \quad (4.7)$$

where Δ is a positive term representing the increase in δ_r due to field couplings created by introducing $s > 0$. The variable η no longer cycles about zero, and the system is slowly pushed away from stability, and enters a pseudo-cyclic phase in which a series of bounces occur with increasing local minima and maxima of the expansion volume, ab^2 , as shown in figure 3. The behaviour of ϵ 4.2 is affected by this expansion (recall that ϵ is already small). The expansion of $b(t)$ means that H_b is no longer small, and the equation gains a damping term. Similarly, the frequency of oscillations, $\sqrt{2}/b$, is reduced by this expansion in $b(t)$ and the solution takes the approximate form of a damped harmonic oscillator. Note that in the derivation of 4.2 only the smallness of ϵ was used - therefore this damping behaviour is present in all expanding solutions.

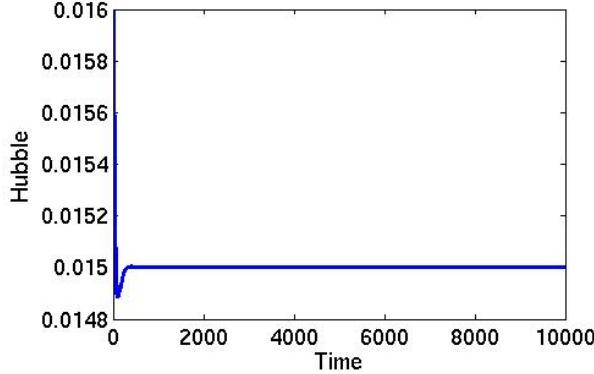


FIG. 4: Hubble expansion rate versus time for an inflating solution. Initial values: $\beta_0 = 0.03$, $\beta_H = \beta_\sigma = 0$, $r = 1$, $\sigma = 5 \times 10^{-4}$, $a = 10^4$, $H = 1.6 \times 10^{-2}$.

B. Ghost-induced Inflation

Spacetimes which exhibit inflation are of special interest to cosmologists because inflation can solve a number of well known puzzles about the universe's structure [29], and make a series of detailed predictions that can be tested by observations of the microwave background radiation [30]. Typical inflationary models exhibit expansion in which the Hubble parameter is (approximately) constant for a finite time interval. In general relativistic cosmology this is usually achieved by introducing a matter content that is (or is equivalent to) a scalar field subject to a self-interaction potential whose contribution to the total energy density is dominant during this expansion, with an equation of state close to that of that produced by an exact cosmological constant with $p_\Lambda = -\rho_\Lambda$.

Inflation induced by ghost fields has been studied as an alternative to the usual slow-roll models [31]. Such models have potentially observable consequences for the microwave background trispectrum [32] [33], but require that the translation invariance of the scalar ghost field is broken. In the BSBM models under consideration, however, translation invariance can be preserved, with the field coupling responsible for creating the inflationary energy density.

Let us examine the case of a linear coupling-induced inflation with $s = \beta_0 \rho_\psi$, for constant $\beta_0 > 0$. When the volume is large there exists an asymptotic solution of the form

$$\rho_r = \frac{9\beta_0^2}{4}, \quad \rho_\psi = -\frac{3\beta_0^2}{2}, \quad H = -\frac{\beta_0}{2}, \quad (4.8)$$

in the isotropic case. This solution is stable, and is approached by dynamical trajectories, as shown in figure 4. Under a small perturbation $H = -\beta_0/2 + h$ and $r = 1 + \epsilon$ we find that to first order in the small parameters:

$$\dot{h} = -\beta_0 h \quad (4.9)$$

$$\dot{\epsilon} = -3\dot{\epsilon}H - 2\epsilon/b \quad (4.10)$$

In this solution, $b(t)$ is exponentially growing, so the final term in 4.9 quickly becomes negligible. Therefore, although a shearing expansion may occur, $\dot{\epsilon}$ quickly falls to zero, locking the shear at a fixed value. This inflationary phenomenon is not unique to linear couplings with $\beta_H = \beta_\sigma = 0$, but is simplest to demonstrate in this case. Likewise, there is no requirement for the matter field to consist solely of radiation - introducing more matter fields with couplings whose sign is determined to be in accordance with the second law of thermodynamics yields a system which also exhibits inflation of this type. For most couplings, however, the inflationary phase will end when the dust field becomes dominant.

The evolution of the fine structure constant is shown in figure 5. Initially, the solution is like an ascending staircase with rapid changes at each scale factor bounce, see for comparison [10]. Monotonicity of α is ensured since the scalar field cannot have positive energy density; since $\dot{\psi} = \sqrt{\frac{-2\rho_\psi}{w}}$, we have $\dot{\psi} \geq 0$ for all time, and so ψ cannot oscillate through maxima and minima. Across repeated bounces, $\log(\alpha)$ will appear to increase in steps when $|\rho_\psi|$ is small as the relative size of this energy density oscillates greatly within a single cycle. However, as energy is transferred into the radiation field, these steps will become less apparent, eventually approaching a constant gradient once the ghost field reaches the condition for de Sitter inflation to occur. Thus, even though the universe oscillates from cycle to cycle, the fine structure 'constant' continues increasing from cycle to cycle

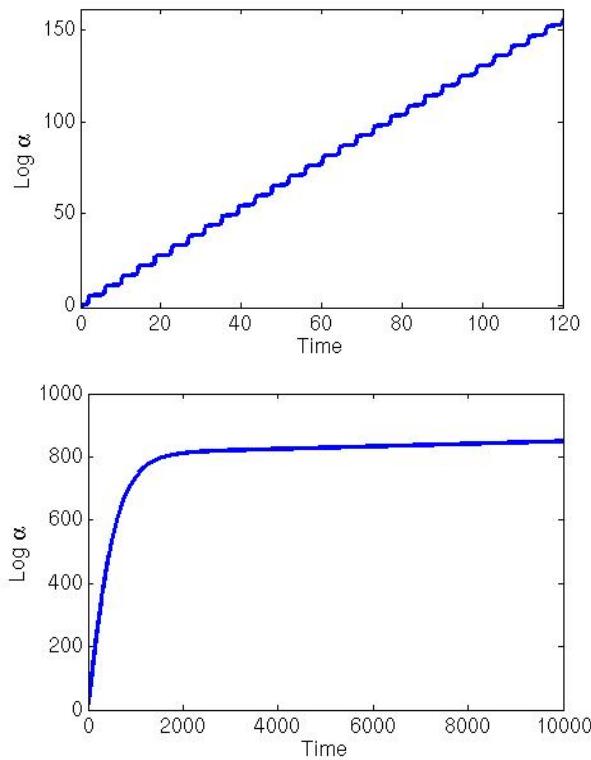


FIG. 5: Evolution of $\log(\alpha)$ versus time shown over a short timescale (top), and the same evolution over a long timescale (bottom). Initial values: $\beta_0 = 0.01$, $\beta_H = \beta_\sigma = 0$, $r = 1.01$, $\sigma = 0$, $a = 1$, $H = 0$, $\rho_\psi = -0.01$.

and there will typically only be a finite interval of cycles in which α takes values that allow stable atoms to exist [34],[35].

V. DISCUSSION

In this paper, we examined the new effect of introducing anisotropies into the BSBM framework for varying α , although the conclusions have broader applicability to anisotropic cosmologies containing ghost fields and entropy-increasing energy exchanges between fields. In particular, we studied the dynamics of locally rotationally symmetric Bianchi IX cosmologies. It was shown that under certain conditions the bouncing behaviour observed in isotropic models persists, with the fine structure constant" changing in an almost step-like increasing manner between cycles as time increases. It is apparent from 3.2 that on short scales there is a tension

between the shear terms and ghost field, as both scale as the inverse square of the volume. When the anisotropy is small, the contribution from the ghost field dominates. This leads to a bouncing model, reproducing closely the results seen in [10]. Furthermore, there exists a static solution, perturbations about which lead to a sequence of anisotropic bouncing phases. When there is a coupling between the matter fields, the second law of thermodynamics ensures that this process isotropizes the system by energy exchange.

The resulting dynamics lead to a pseudo-cyclic universe in which the fine structure constant monotonically increases across bounces, and for small values of the associated dielectric scalar field, this increase is dominated by dynamics near the bounce point. The minimum and maximum volumes of the universe also increase across cycles, with the total energy density decreasing. Eventually, the model reaches a point at which the coupling between fields fixes the energy density to be constant in time, and the universe undergoes a de Sitter phase in which it inflates. Since this model is limited to include only the dielectric field and radiation, there is no transition to the dust-dominated era that one would expect at the end of this phase, and so inflation is endless within the model. It is possible to find solutions with dust in which the system again reaches a point of steady inflation. However within the space of couplings with $s > 0$ which obey the second law, these solutions are a set of measure zero.

The evolution of α throughout the history of BSBM universes displays interesting traits. At late times, in a large universe it will appear that α has settled to a constant value. In doing so, throughout a series of oscillations the universe will have isotropised greatly, with α stepping up between cycles. As the dynamics are invariant under changing the initial value of α there is no obvious mechanism to determine the constant to which it will approach.

Acknowledgements

DS acknowledges support from a Templeton Foundation grant.

[1] A. D. Rendall, *The Nature of spacetime singularities* (World Scientific, Washington, D.C., 2005).

[2] L. Andersson and A. D. Rendall, Commun. Math. Phys. **218**, 479 (2001).

[3] D. Garfinkle, Class. Quant. Grav. **29**, 244003 (2012).

[4] J. Bekenstein, Phys. Rev. **D25**, 1527 (1982).

[5] H. B. Sandvik, J. D. Barrow, and J. Magueijo, Phys. Rev. Lett. **88**, 031302 (2002).

[6] J. Webb, J. King, M. Murphy, V. Flambaum, R. Carswell, et al., Phys. Rev. Lett. **107**, 191101 (2011).

[7] J. D. Barrow and S. Z. Lip, Phys. Rev. **D85**, 023514 (2012).

[8] J. D. Barrow and B. Li, Phys. Rev. **D78**, 083536 (2008).

[9] C. Brans and R. Dicke, Phys. Rev. **124**, 925 (1961).

[10] J. D. Barrow, D. Kimberly, and J. Magueijo, Class. Quant. Grav. **21**, 4289 (2004).

[11] J. D. Barrow and C. G. Tsagas, Class. Quant. Grav. **26**, 195003 (2009).

[12] G. Gibbons (2003), hep-th/0302199.

[13] R. C. Tolman, *Relativity, Thermodynamics, and Cosmology* (Clarendon Press, Oxford, 1934).

[14] G. Lemaître, Ann. de la Soc. Scientifique de Bruxelles **A53**, 51 (1933).

[15] J. D. Barrow and M. P. Dąbrowski, Mon. Not. R. Astron. Soc. **275**, 850 (1995).

[16] A. Ashtekar and P. Singh, Class. Quant. Grav. **28**, 213001 (2011).

[17] R. Brandenberger, Phys. Rev. **D80**, 043516 (2009).

[18] Y. Misonoh, K.-i. Maeda, and T. Kobayashi, Phys. Rev. **D84**, 064030 (2011).

[19] S. Calogero and J. M. Heinzle, Physica **D240**, 636 (2011).

[20] T. Damour, in *Neutrino telescopes. Proceedings*, edited by M. Baldo-Ceolin (Instituto Veneto Di Scienze, Venice, 2003), pp. 595–609.

[21] J.-P. Uzan, Rev. Mod. Phys. **75**, 403 (2003).

[22] E. W. Kolb, M. J. Perry, and T. Walker, Phys. Rev. **D33**, 869 (1986).

[23] J. D. Barrow, Phys. Rev. **D71**, 083520 (2005).

[24] H. Marion, F. Pereira Dos Santos, M. Abgrall, S. Zhang, Y. Sortais, et al., Phys. Rev. Lett. **90**, 150801 (2003).

[25] J. Berengut and V. Flambaum, Europhys. Lett. **97**, 20006 (2012).

[26] D. Shaw and J. Barrow, Phys. Rev. **D73**, 123505 (2006). D. Shaw and J. Barrow, Phys. Rev. **D73**, 123506 (2006).

[27] J. D. Bekenstein and M. Schiffer, Phys. Rev. **D80**, 123508 (2009).

[28] A. H. Taub, Ann. Math. **53**, 472 (1951).

[29] A. Linde, Mod. Phys. Lett. **A1**, 81 (1986).

[30] P. A. R. Ade et al. (Planck Collaboration) (2013), arXiv:1303.5062 [astro-ph.CO].

[31] N. Arkani-Hamed, P. Creminelli, S. Mukohyama, and M. Zaldarriaga, JCAP **0404**, 001 (2004).

[32] K. Izumi and S. Mukohyama, JCAP **1006**, 016 (2010).

[33] Q.-G. Huang, JCAP **1007**, 025 (2010).

[34] J. D. Barrow and F. J. Tipler, *The Anthropic Cosmological Principle* (Oxford University Press, Oxford, 1986).

[35] E. H. Lieb, Rev. Mod. Phys. **48**, 553 (1976).